Point Estimate Method

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Point Estimate Method

❖ Hong's point estimate methods

- To account for uncertainties, that may arise from different sources as load demand or generation unit outages.

- Many engineering problems are subject to uncertainty due to:
  - The inherent randomness of natural phenomena
  - The implicit and inaccurate assumptions related to the considered modeling approach.

Point Estimate Method

- **In power flow problem**

- The information is not certain, and for this reason, it is desirable to assess system variables (bus voltages and line flows) for a range of load and generation conditions.

- **There are several techniques to deal with problems under uncertainty.**
- **These techniques may be classified into the three main categories:**
  - Monte Carlo simulation,
  - Analytical methods
  - and approximate methods.

Monte Carlo simulation

❖ Monte Carlo simulation

- Monte Carlo simulation randomly generates values for uncertain input variables, and these values are taken into account to solve a deterministic problem.

- This technique has been widely used in power systems analysis to model uncertainty.

- The main drawback of the Monte Carlo method is the great number of simulations required to attain convergence

Analytical methods

❖ Analytical methods

- Analytical methods are computationally more effective, but they require some mathematical assumptions in order to simplify the problem.

Approximate methods

❖ Approximate methods

- Approximate methods provide an approximate description of the statistical properties of output random variables.
- First-order second-moment method (FOSMM)
- point estimate methods.

Approximate methods

❖ Approximate methods

The main advantages are as follows.

1) As Monte Carlo simulation, point estimate methods use deterministic routines for solving probabilistic problems;

2) However, they require a much lower computational burden.

3) Furthermore, point estimate methods overcome the difficulties associated with the lack of perfect knowledge of the probability functions of stochastic variables

Moments

❖ Moments

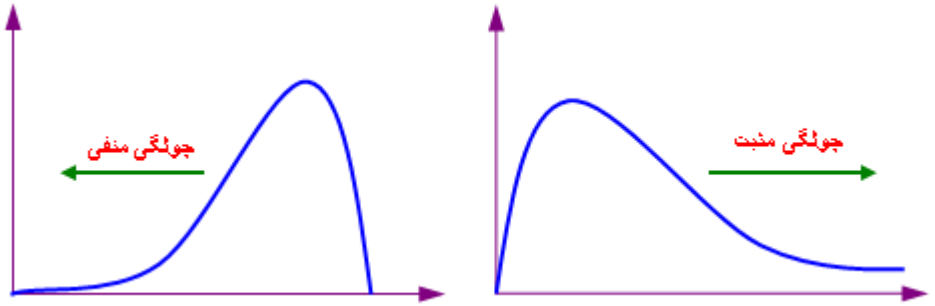
- since these functions are approximated using only their first few statistical moments

- mean,
- variance,
- skewness, probability distribution of a real-valued random variable "leans" to one side of the mean
- Kurtosis: measure of the "peakedness" of the probability distribution of a real-valued random variable

Moments

❖ Moments

- Skewness:



- Probability distribution of a real-valued random variable "leans" to one side of the mean

$$\gamma_1 = \mathrm{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} = \frac{\mathrm{E}\left[(X-\mu)^3\right]}{\left(\mathrm{E}\left[(X-\mu)^2\right]\right)^{3/2}} = \frac{\kappa_3}{\kappa_2^{3/2}},$$

- Kurtosis:

$$\beta_2 = \frac{\mathrm{E}[(X-\mu)^4]}{(\mathrm{E}[(X-\mu)^2])^2} = \frac{\mu_4}{\sigma^4} \qquad \gamma_2 = \frac{\kappa_4}{\kappa_2^2} = \frac{\mu_4}{\sigma^4} - 3$$

Properties of Point Estimates

❖ Properties of Point Estimates

- **Unbiased Estimates**

  - Definitions

  - A point estimate $\hat{\theta}$ for a parameter θ is said to be unbiased if

$$E(\hat{\theta}) = \theta$$

  - If a point estimate is not unbiased, then its bias is defined to be

$$bias = E(\hat{\theta}) - \theta$$

Comparison of PEM methods

❖ Comparison of PEM methods

TABLE I
QUALITATIVE DESCRIPTION OF POINT ESTIMATE METHODS

Method's author [Ref.]	Number of simulations	Efficiency in Large Scale problems	Ability to handle	
			correlated variables	asymmetric variables
Rosenblueth [19]	2^m	Very Low	Yes	Yes
Li [20]	$O(m^3)$	Low	Yes	Yes
Harr [24]	$2m$	High	Yes	No
Hong [25]	Km or $Km+1$	High	No	Yes

[19] E. Rosenblueth, "Two-point estimates in probability," *Appl. Math. Model.*, vol. 5, pp. 329–335, Oct. 1981.

[20] K. S. Li, "Point-estimate method for calculating statistical moments," *J. Eng. Mech.-ASCE*, vol. 118, no. 7, pp. 1506–1511, 1992.

[24] M. E. Harr, "Probabilistic estimates for multivariate analysis," *Appl. Math. Model.*, vol. 13, no. 5, pp. 313–318, 1989.

[25] H. P. Hong, "An efficient point estimate method for probabilistic analysis," *Reliab. Eng. Syst. Saf.*, vol. 59, pp. 261–267, 1998.

Hong's Point Estimate Method

❖ Hong's Point Estimate Method

- K points for each random variables, named concentrations
- By using these points and the function F, which relates input and output variables, information about the uncertainty associated with problem output random variables can be obtained.

Point Estimate Method

❖ Point Estimate Method

- The aim of any PEM is to compute the moments of a random variable Z that is a function of M random input variables

$$z = F(p_1, p_2, \ldots, p_M)$$

Point Estimate Method

❖ Point Estimate Method

The Kth concentration ($p_{m,k}$, $w_{m,k}$) of random variable p_m can be defined as a pair composed of a location $p_{m,k}$ and a weight $w_{m,k}$

$$k=1, 2, \ldots, K$$

F has to be evaluated only K times for each input random variable p_m at the points made up of the Kth location $p_{m,k}$ of the input random variable p_m and the mean of the M-1 remaining input variables, i.e. .

$$(\mu_{p_1}, \mu_{p_2}, \ldots, p_{m,k}, \ldots, \mu_{p_M})$$

Point Estimate Method

❖ Point Estimate Method

The number K of evaluations depends on the scheme used.

Total simulation number:

$$K \times M$$

$$(\mu_{p_1}, \mu_{p_2}, ..., p_{m,k}, ..., \mu_{p_M})(K \times M)$$

$$+$$

$$(\mu_{p_1}, \mu_{p_2}, ..., \mu_{p_m}, ..., \mu_{p_M})$$

$$K \times M + 1$$

Point Estimate Method

❖ Point Estimate Method

- The location to be determined

$$p_{m,k} = \mu_{p_m} + \xi_{m,k}.\sigma_{p_m}$$

$$m = 1,...,M$$

$$k = 1,...,K$$

❑ Standard location

K unknown ξ for each p_m

$$\xi_{m,k} = ?$$

$$m = 1,...,M$$

$$k = 1,...,K$$

Point Estimate Method

❖ Point Estimate Method

❑ Standard location

K unknown w for each p_m

$$w_{m,k} = ?$$

$$m = 1,\ldots,M$$

$$k = 1,\ldots,K$$

- Unknown parameters
- Known equations

$$2 \times K \times M$$

Point Estimate Method

❖ Nonlinear system of equations

- For each m

$$\begin{cases} \sum_{k=1}^{K} w_{m,k} = \dfrac{1}{M} \\ \sum_{k=1}^{K} w_{m,k} \left(\xi_{m,k}\right)^{j} = \lambda_{m,j} \qquad j = 1,\ldots,2K-1 \end{cases}$$

Point Estimate Method

❖ Calculation of the moments

j^{th} Moment

$$\lambda_{m,j} = \frac{\int_{-\infty}^{+\infty} \left(p_m - \mu_{p_m} \right)^j f_{p_m} dp_m}{\sigma_{p_m}^{\ j}}$$

$$p_{m,k} = \mu_{p_m} + \xi_{m,k} . \sigma_{p_m}$$

$$(\mu_{p_1}, \mu_{p_2}, ..., p_{m,k}, ..., \mu_{p_M})$$

$$Z(m,k) = F(\mu_{p_1}, \mu_{p_2}, ..., p_{m,k}, ..., \mu_{p_M})$$

$$E\left[Z^j \right] = \sum_{m=1}^{M} \sum_{k=1}^{K} w_{m,k} \left(Z(m,k) \right)^j$$

$$\mu_Z = E\left[Z \right]$$

$$\sigma^2 = E\left[Z^2 \right] - E\left[Z \right]^2$$

Point Estimate Method

❖ Solving nonlinear equations system

- The analytical solution exists for 2m method (K=2)
- m is the number of input random variables

$$\xi_{l,1} = \frac{\lambda_{l,3}}{2} + \sqrt{m + \left(\frac{\lambda_{l,3}}{2}\right)^2} \quad \xi_{l,2} = \frac{\lambda_{l,3}}{2} - \sqrt{m + \left(\frac{\lambda_{l,3}}{2}\right)^2}$$

$$w_{l,1} = -\frac{1}{m}\frac{\xi_{l,2}}{\xi_{l,1} - \xi_{l,2}} \quad w_{l,2} = \frac{1}{m}\frac{\xi_{l,1}}{\xi_{l,1} - \xi_{l,2}}$$

Point Estimate Method

❖ Solving nonlinear equations system

2m+1

K=3, $\xi_{l,3} = 0$

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k} \sqrt{\lambda_{l,4} - \frac{3}{4}\lambda_{l,3}^2} \quad k = 1,2 \quad \xi_{l,3} = 0$$

$$w_{l,k} = \frac{(-1)^{3-k}}{\xi_{l,k}\left(\xi_{l,1} - \xi_{l,2}\right)} \quad k = 1,2$$

$$w_{l,3} = \frac{1}{m} - \frac{1}{\lambda_{l,4} - \lambda_{l,3}^2} .$$

Point Estimate Method

❖ Solving nonlinear equations system

2m+1

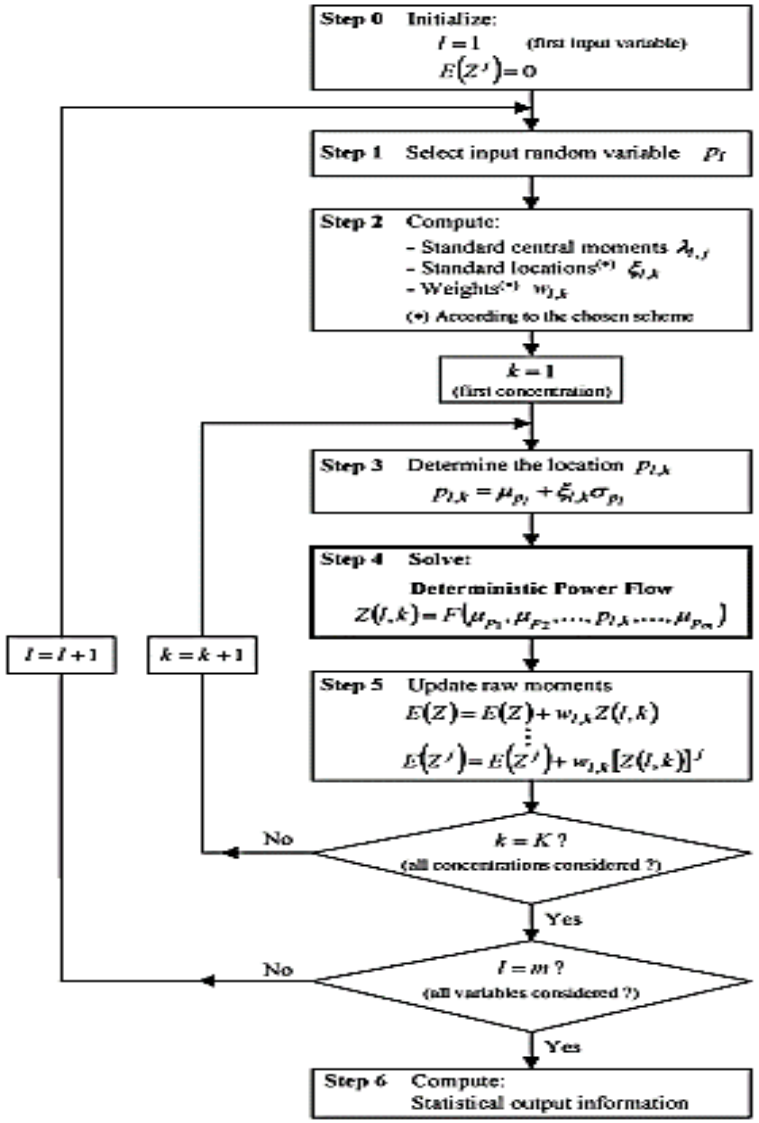
K=3,

$\xi_{l,3} = 0$ → $(\mu_{p_1}, \mu_{p_2}, \ldots, \mu_{p_l}, \ldots, \mu_{p_m})$ are the same

$$w_0 = \sum_{l=1}^{m} w_{l,3} = 1 - \sum_{l=1}^{m} \frac{1}{\lambda_{l,4} - \lambda_{l,3}^2} .$$

Point Estimate Method

- Application of PEM to PPF



Point Estimate Method

❖ Application of PEM to PPF Performance Evaluation

$$\varepsilon_{\mu}^{x} = \left| \frac{\mu_{MC}^{x} - \mu_{PEM}^{x}}{\mu_{MC}^{x}} \right| \times 100 \quad [\%]$$

$$\varepsilon_{\sigma}^{x} = \left| \frac{\sigma_{MC}^{x} - \sigma_{PEM}^{x}}{\sigma_{MC}^{x}} \right| \times 100 \quad [\%]$$

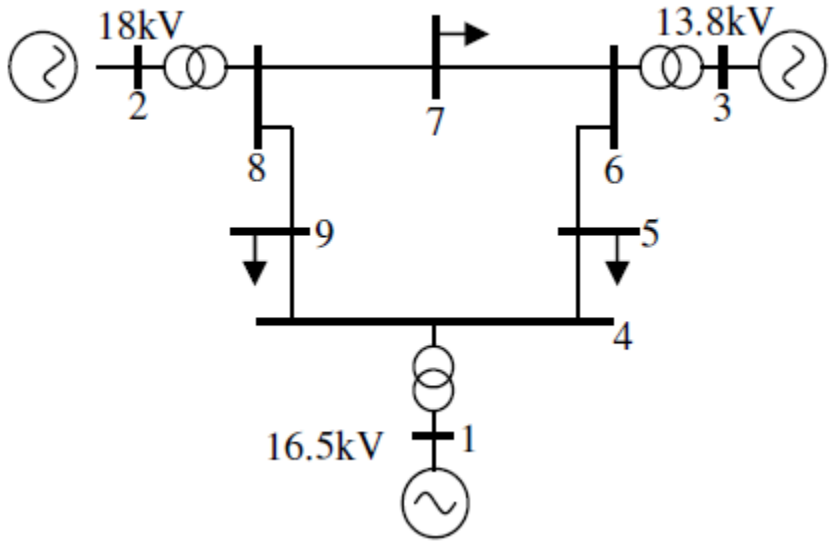
TABLE III
AVERAGE ERROR INDICES FOR IEEE 14-BUS TEST SYSTEM

IEEE 14		V	δ	P_i	Q_i	$P_{i,j}$	$Q_{i,j}$
2m	$\bar{\varepsilon}_{\mu}^{*}$	0.0018	0.0953	0.1664	0.5179	0.0806	0.3272
	$\bar{\varepsilon}_{\sigma}^{*}$	0.5120	0.5101	1.5090	0.8331	0.5361	0.6987
2m+1	$\bar{\varepsilon}_{\mu}^{*}$	0.0018	0.0954	0.1665	0.5192	0.0806	0.3281
	$\bar{\varepsilon}_{\sigma}^{*}$	0.2553	0.5933	1.5506	0.8273	0.5304	0.5312
4m+1	$\bar{\varepsilon}_{\mu}^{*}$	0.0018	0.0954	0.1665	0.5192	0.0806	0.3281
	$\bar{\varepsilon}_{\sigma}^{*}$	0.2552	0.5933	1.5506	0.8273	0.5304	0.5312

- MC with 10000 runs

Point Estimate Method

❖ **Application of PEM to Probabilistic Small Signal stability**



Bus		Distribution of bus power injection	
No.	Type	Active Power	
		$\overline{P}$	σ_P
5	Load	0.9	0.1
7	Load	1.0	0.4
9	Load	1.25	0.1

Point Estimate Method

❖ Application of PEM to Probabilistic Small Signal stability

No.	Mean-eig $\overline{\lambda}$			α_i^*		ξ_i^* ($\xi_c = 0.1$)	
	TPE	MCS_{2000}	AVER	TPE	MCS_{2000}	TPE	MCS_{2000}
1	0.00018701	0.00018699	0.00018524	-4.6409	-4.6364	$-\infty$	$-\infty$
2	-0.28222	-0.28222	-0.28215	96.62	96.775	$+\infty$	$+\infty$
3	-0.15197-8.7133i	-0.15197-8.7134i	-0.1511-8.7271i	18.814	18.781	-77.388	-77.087
4	-0.15197+8.7133i	-0.15197+8.7134i	-0.1511+8.7271i	18.814	18.781	-77.388	-77.087
5	-0.0947-12.239i	-0.0947-12.239i	-0.094676-12.238i	254.5	255.28	-2523	-2531
6	-0.0947+12.239i	-0.0947+12.239i	-0.094676+12.238i	254.5	255.28	-2523	-2531
7	-24.803-29.576i	-24.803-29.576i	-24.754-29.599i	76.663	76.597	189.11	187.35
8	-24.803+29.576i	-24.803+29.576i	-24.754+29.599i	76.663	76.597	189.11	187.35
9	-44.837	-44.837	-44.922	100.59	99.905	$+\infty$	$+\infty$
10	-40.644-25.82i	-40.644-25.82i	-40.589-25.915i	125.78	124.92	295.42	289.38
11	-40.644+25.82i	-40.644+25.82i	-40.589+25.915i	125.78	124.92	295.42	289.38
12	-73.533	-73.533	-73.601	161.1	160.55	$+\infty$	$+\infty$