University of Tabriz

Faculty of Electrical and Computer Engineering

First HW, Probabilistic Power System Analysis — Due date: Dec 06, 2020

1- Consider the Gaussian distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

a) Show that all odd moments $E[(X-\mu)^n]$ of order n≥3 are zero.

b) Prove that the following relationship

$$Var(X) = E\left[X^2\right] - \mu^2$$

in which μ is the mean of the random variable X, holds true.

2- Given the distribution of a Poisson with variable *n*,

$$P(n) = \frac{\mu^n}{n!} e^{-\mu}$$

Show that the mean is given by μ and that the variance is also given by μ.

3- Consider the following Gaussian dataset in which the dependent variables are assumed to have the same unknown standard deviation σ,

x_i	y_i
0.0	0.0
1.0	1.5
2.0	1.5
3.0	2.5
4.0	4.5
5.0	5.0

The data are to be fit to a linear model.

(a) Using the maximum likelihood method, find the analytic relationships between $\sum x_i$, $\sum y_i$, $\sum x_i y_i$, $\sum x_i^2$, and the model parameters a and b.

(b) Show that the best fit values of the model parameters are $a = 0$ and $b = 1$.

4- Voltage endurance tests have been performed on generator coils. While analyzing test results, an engineer postulates that the variable X representing the voltage endurance of the coils has either a normal PDF N (20, 0.1) (in kV) or a Cauchy PDF,

$$f_X(x) = \frac{1}{0.1\pi\left[1 + \left(\frac{x - 20}{0.1}\right)^3\right]}$$

If he chooses to model X by PDF for which $P\ (|X - 20| < 0.10)$ is maximum, which PDF would be chosen?

5- The following records were obtained for the operating time (in years) of a distribution transformer, which has experienced thirteen outages during the period of operation.

2244, 2340, 2436, 2586, 2638, 3018, 3325, 3504, 3969, 4024

Applying the Kolmogorov-Smirnov test, determine at the 5% significance level the relative goodness of fit between the following distributions:

(a) A Poisson distribution.

(b) A normal distribution N (2000, 500).

(c) A lognormal distribution with $\mu = 3000$ and $\sigma = 510$.

(d) An exponential distribution.