University of Tabriz

Faculty of Electrical and Computer Engineering

Third HW, Power system optimization — Due date: May 30, 2019

1- Consider the following problem:

$$\underset{x_1, x_2}{\text{minimize}} \qquad z = (x_1 - 4)^2 + (x_2 - 1)^2$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\geq 6 \\ -x_1^2 + x_2 &\geq -2\ . \end{aligned}$$

a) Find the optimal solution for this primal problem geometrically.
b) Obtain graphically the optimal multipliers values associated with both constraints, i.e., the optimal solution of the dual problem.
c) Verify that KKT conditions hold.
d) Formulate and solve the dual problem and verify the objective function of primal and dual is equal.

2- Find the minimum length of *a* ladder that must lean against a wall if a box of dimensions *a* and *b* is placed right at the corner of that same wall (Fig. 1). Formulate and solve the primal and dual problems.

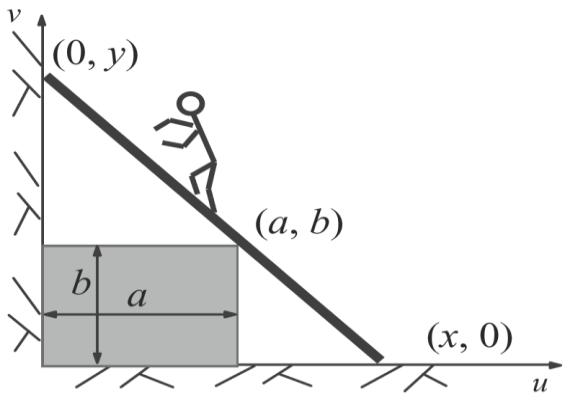

Fig. 1 Illustration of the ladder problem

3- Using the KKT conditions find the possible candidates (x1, x2) and (λ, μ1, μ2, μ3) for solving the following problem:

$$\underset{x_1, x_2}{\text{minimize}} \qquad Z = -x_1 + x_2$$

subject to

$$\begin{aligned} -x_1^2 + x_2 = 0 \ &: \ \lambda \\ x_1^2 + x_2^2 - 4 \leq 0 \ &: \ \mu_1 \\ -x_1 \leq 0 \ &: \ \mu_2 \\ -x_2 \leq 0 \ &: \ \mu_3 \ , \end{aligned}$$

and its dual, where λ, μ1, μ2, and μ3 are the dual variables.

4- For the following problem:

$$\underset{x1,x2}{Minimize} \quad -X_1 + 3X_2^2$$

s.t

$$(1-X_2)^3 - X_1 \geq 0$$
$$X_1 + X_2 - 1 \geq 0$$
$$X_1, X_2 \geq 0$$

a) Plot the feasible region and find the optimal solution.

b) Find the optimal solution using the KKT conditions. Find the values of Lagrange multipliers at optimal solution.
c) Compute the Hessian of Lagrangian function at the obtained optima point.
d) Formulate and solve the dual problem and verify the objective function of primal and dual is equal.

5- Two thermal units have the following characteristics:

Fuel cost functions:

$C_1(P_1) = 91 + 7.4P_1 + 0.031P_1^2$ \$/MWh

$C_2(P_2) = 121.8 + 7.8P_2 + 0.048P_2^2$ \$/MWh

Emission functions:

$E_1(P_1) = 15 + 3.4P_1 + 0.0092P_1^2$ kg/hr

$E_2(P_2) = 17.5 + 3.76P_2 + 0.0057P_2^2$ kg/hr

$80 \le P_1 \le 400$

$60 \le P_2 \le 300$

The total load is 580 MW.

a) Find the production of each unit to minimize the total cost.

b) Find the production of each unit to minimize the total emission.

c) Find the production of each unit to minimize cost and emission simultaneously using epsilon constraint method. Use fuzzy satisfying method to select the best compromise solution. Two objectives are equally important.

6- Write and prove weak duality theory for an inequality constrained NLP.

$$\begin{aligned} z^* = \text{minimum}_x \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \le 0, \quad i = 1, \ldots, m \\ & x \in X, \end{aligned}$$

7- Solve the following bi-level problem using KKT method.

$$\underset{X,Y}{Minimize} \quad -X + 3Y$$

s.t

$$-X + 8Y - 10 \le 0$$

$$X + 4Y \ge 7$$

where Y, for each value of X, is the solution of:

$$\underset{Y}{Minimize} \quad X + 2Y$$

s.t

$$X - Y \le 0$$

$$-X - 3Y \le 4$$

Change the upper level constraint to the following and solve it using dual method.

$$-X + 3Y - 4 \le 0$$

$$-Y + \frac{3}{2} \le 1$$

8- Solve the following bi-level problem using KKT method.

$$\underset{x1}{Minimize} \quad -X_1 + 2X_2$$

s.t

$$2 \le X_1 \le 9$$

$$\underset{x2}{Minimize} \quad 4 - 3X_2$$

s.t

$$(1 + 7X_2) - 6X_1 \ge 0$$

$$X_1 + 2X_2 \le 19$$

9- Solve the following optimization problem using "GA" in Matlab. Place screen shots of the code and results in your answer sheet.

$$\underset{x1,x2}{Minimize} \quad X_1^3 + 1.5X_2^3$$

s.t

$$X_1 + 3X_2 \ge 0$$

$$2X_1 + X_2^2 \ge 7$$

$$X_1^2 + X_2^2 \le 5$$

$$-2 \le X_1, X_2 \le 3$$