University of Tabriz

Faculty of Electrical and Computer Engineering

Third HW, Optimization in Power Systems Course   Due date: June 22, 2017

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1. *Does the point(x1,x2)=(1,14) satisfy the Karush-Kuhn-Tucker conditions for the following problem?*

$$\text{minimize} \qquad -x_1 - x_2$$

$$\text{s.t.} \qquad -x_1^3 + 6x_1^2 - 9x_1 + x_2 - 10 \leq 0$$

$$-x_2 + 14 \leq 0$$

$$x_1 - 5 \leq 0.$$

2. *Find KKT points of the following problem:*

$$\text{Minimize } f(x, y) = e^{-x-y}$$

Subject to

$$e^x + e^y \leq 20$$

$$x \geq 0,$$

3. Build dual of the following linear programming problem using the method of dual of an NLP problem.

$$\text{minimum}_x \quad c^T x$$

$$\text{s.t.} \quad Ax \geq b$$

4. Find dual of the generic quadratic programming problem. Suppose Q is positive definite.

$$\text{minimum}_x \quad \tfrac{1}{2}x^T Q x + c^T x$$

$$\text{s.t.} \quad Ax \geq b$$

5. Write and prove weak duality theory for an inequality constrained NLP.

$$
\begin{aligned}
z^* = \text{minimum}_x \quad & f(x) \\
\text{s.t.} \quad & g_i(x) \quad \leq \quad 0, \quad i = 1, \ldots, m \\
& x \in X,
\end{aligned}
$$

6. For the following problem:

$$\min_{x_1, x_2} -x_1 + x_2^2 \text{ subject to } \begin{cases} (1 - x_1)^3 - x_2 & \geq \quad 0 \\ x_1 + x_2 - 1 & \geq \quad 0 \end{cases}$$

   a. Plot the feasible region and find the optimal solution.
   b. Find the optimal solution using the KKT conditions. Find the values of Lagrange multipliers at optimal solution.
   c. Compute the Hessian of Lagrangian function at the obtained optima point.

7. For the following problem:

$$\min_{x_1, x_2} \frac{1}{2}\alpha x_1^2 + \frac{1}{2}x_2^2 + x_1 \text{ subject to } x_1 \geq 1.$$

a. For $\alpha = 1$, find the optimal solution of the primal. Formulate and solve the dual problem and verify the objective function of primal and dual is equal.
b. Repeat a. for $\alpha = 0$

8. Solve the following optimization problem using "GA" in Matlab. Place screen shots of the code and results in your answer sheet.

$$\begin{aligned}
\underset{\mathbf{x}\in\mathbb{R}^2}{\text{minimize}} \quad & \mathbf{x}_1^3 + \mathbf{x}_2^3 \\
\text{subject to} \quad & \mathbf{x}_1 + 5\mathbf{x}_2 \geq 0 \\
& \mathbf{x}_1^2 + \mathbf{x}_2^2 \leq 2 \\
& -2 \leq \mathbf{x} \leq 2
\end{aligned}$$

9. *Solve the following bilevel problem using KKT method.*

$$\begin{aligned}
\text{minimize}_{x,y} \quad & x - 2y \\
\text{subject to} \quad & -x + 3y - 4 \leq 0,
\end{aligned}$$

where y, for each value of x, is the solution of:

$$\begin{aligned}
\text{minimize}_{y} \quad & x + y \\
\text{subject to} \quad & x - y \leq 0, \\
& -x - y \leq 0.
\end{aligned}$$

Change the upper level constraint to the following and solve it using dual method.

$$\begin{aligned}
-x + 3y - 4 &\leq 0, \\
-y + \tfrac{1}{2} &\leq 0,
\end{aligned}$$

10. *Solve the following bilvel problem using KKT method.*

$$\min_{x} x + 3y$$
$$s.t.\ 1 \leq x \leq 6,$$
$$\min_{y} -y$$
$$s.t.\ \ x + y \leq 8,$$
$$x + 4y \geq 8,$$
$$x + 2y \leq 13$$

11. *Two thermal units have the following characteristics:*

Fuel cost functions:

$$C_1(P_1) = 83 + 6.2P_! + 0.021P_1^2 \qquad \$/\mathrm{MWh}$$
$$C_2(P_2) = 120.75 + 7.5P_2 + 0.045P_2^2 \qquad \$/\mathrm{MWh}$$

Emission functions:

$$E_1(P_1) = 0.009\ P_1^{\ 2} + 3.0\ P_1 + 12\ \ kg\ /\ hr$$
$$E_2(P_2) = 0.005\ P_2^{\ 2} + 3.89\ P_2 + 14\ \ kg\ /\ hr$$

$$80 \leq P_1 \leq 400$$
$$60 \leq P_2 \leq 300$$

The total load is 560 MW.

a) Find the production of each unit to minimize the total cost.
b) Find the production of each unit to minimize the total emission.

c) Find the production of each unit to minimize cost and emission simultaneously using epsilon-constraint method. Use fuzzy satisfying method to select the best compromise solution. Two objectives are equally important.