University of Tabriz

Faculty of Electrical and Computer Engineering

Second HW, Optimization of Power Systems Course   Due date: May 13, 2017

1. For the following problem:

$$\underset{x_1, x_2, x_3, x_4}{\text{minimize}} \quad z = -2x_1 - x_2 - x_3 + x_4$$

subject to

$$\begin{array}{rrrrl}
x_1 & + 2x_2 & & & \leq 5 \\
 & & -x_3 & + x_4 & \leq 2 \\
 & & 2x_3 & + x_4 & \leq 6 \\
x_1 & & + x_3 & & \leq 2 \\
x_1 & + x_2 & & + 2x_4 & \leq 3 \\
 & & & x_1, x_2, x_3, x_4 & \geq 0\,.
\end{array}$$

1. Check that the following vector (x_1, x_2, x_3, x_4) is a solution:

$$x_1 = 1, \;\; x_2 = 2, \;\; x_3 = 1, \;\; x_4 = 0, \;\; z = -5.$$

2. Using the Dantzig-Wolfe decomposition algorithm and by minimizing the objective functions,

$$\begin{array}{rrrrr}
z_1 = & -x_1 & -x_2 & & + x_4 \\
z_2 = & x_1 & + x_2 & -x_3 & \\
z_3 = & x_1 & & -x_3 & + x_4 \\
z_4 = & 2x_1 & + x_2 & & +3x_4\,,
\end{array}$$

obtain the two different feasible solutions (x_1, x_2, x_3, x_4) of the relaxed problem and the associated values of r_i and z, shown in Table 2.5.

Table 2.5. Initial solutions for the subproblems and new added solutions using the Dantzig-Wolfe decomposition algorithm for Exercise 2.2

Iteration	Bounds		Initial solutions for the subproblems						
ν	Lower	Upper	$x_1^{(\nu)}$	$x_2^{(\nu)}$	$x_3^{(\nu)}$	$x_4^{(\nu)}$	$r_1^{(\nu)}$	$r_2^{(\nu)}$	$z^{(\nu)}$
0–1	$-\infty$	∞	5.00	0.00	0.00	0.00	5.00	5.00	−10.00
0–2	$-\infty$	∞	0.00	0.00	3.00	0.00	3.00	0.00	−3.00
			Subproblem solutions						
1	−42.50	17.00	0.00	2.50	0.00	0.00	0.00	2.50	−2.50
2	−5.00	−5.00	–	–	–	–	–	–	–

Iteration	Bounds		Master solutions						
ν	Lower	Upper	$u_1^{(\nu)}$	$u_2^{(\nu)}$	$u_3^{(\nu)}$	$\lambda_1^{(\nu)}$	$\lambda_2^{(\nu)}$	$\sigma^{(\nu)}$	Feasible
1	$-\infty$	17.00	0.00	1.00	0.00	−20.00	0.00	57.00	No
2	−42.50	−5.00	0.30	0.10	0.50	−1.00	−1.00	0.00	Yes

3. Show that using the Dantzig-Wolfe decomposition algorithm the following solution is obtained

$$x_1 = 1.6, \quad x_2 = 1.4, \quad x_3 = 0.4, \quad x_4 = 0, \quad z = -5 \; .$$

4. Compare and discuss the resulting solution and that given in item 1 above.

2.

Consider the hydroelectric river system depicted in Fig. 2.8. The system should be operated so that the demand for electricity is served in every time period of the planning horizon in such a way that total cost is minimum. Data is provided in the Tables 2.6, 2.7, and 2.8. The conversion factor is used to convert water discharge volume to energy.

Solve this multiperiod operation planning problem using the Dantzig-Wolfe decomposition so that the problem decomposes by reservoir.

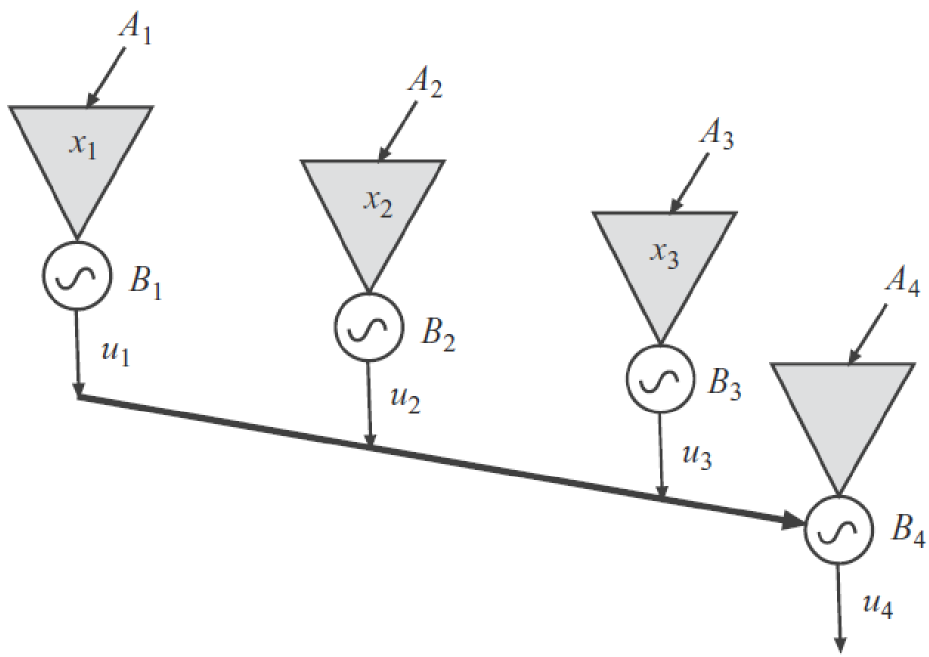

Fig. 2.8. Hydroelectric river system for Exercise 2.7

Table 2.6. Hydroelectric plant data for Exercise 2.7

Hydro plant data				
Unit	1	2	3	4
Initial Volume (hm^3)	104	205	55	0
Maximum Volume (hm^3)	1,000	1,000	1,000	0
Minimum Volume (hm^3)	0	0	0	0
Maximum Discharge (hm^3/h)	30	30	30	80
Minimum Discharge (hm^3/h)	0	0	0	0
Cost ($)	20	10	5	0
Conversion Factor (MWh/hm^3)	10	10	10	10

Table 2.7. Inflow to reservoirs for Exercise 2.7

Inflow to reservoirs (hm^3)				
Reservoir	1	2	3	4
Period 1	35	25	20	10
Period 2	36	26	21	9
Period 3	37	27	22	8
Period 4	36	26	21	7
Period 5	35	25	20	6

Table 2.8. Demand data for Exercise 2.7

Demand data					
Hour	1	2	3	4	5
Demand (MWh)	1,000	1,200	1,300	1,400	1,200

For more detail on problem formulation you can refer to chapter 1 of first reference.

3. Solve the following problem using Benders decomposition.

$$\max_{y_1, y_2, y_3, x_1} \quad y_1 + 3y_2 + y_3 + 4x_1$$

subject to

$$\begin{array}{rrrrcr}
-y_1 & & & + x_1 & \leq & 1 \\
 & 2y_2 & & + 2x_1 & \leq & 4 \\
 & & & x_1 & \leq & 4 \\
 & & & 2x_1 & \leq & 6 \\
 & & & -x_1 & \leq & -1 \\
2y_1 & + y_2 & + 2y_3 & + 2x_1 & \leq & 9 \\
 & & & y_1, y_2, y_3, x_1 & \geq & 0 .
\end{array}$$

Find the optimal solution considering x_1 as a complicating variable.

4-

A businessperson owns two electric lamp warehouses containing respectively 1200 and 1000 lamps. The businessperson serves 3 markets whose demands are respectively 100, 700, and 500 lamps. Transportation costs are given in the table below.

	Market 1	Market 2	Market 3
Warehouse 1	14	13	11
Warehouse 2	13	13	12

How many lamps should the businessperson send from each warehouse to each market so that his benefit is maximum? Solve the problem using the Branch–bound algorithm.

5- Maximize the following objective function using branch and bound method.

$$Z = 120x_1 + 80x_2$$

$$
\begin{array}{rrcl}
2x_1 & +x_2 & \leq & 6 \\
7x_1 & +8x_2 & \leq & 28 \\
x_1 & & \geq & 0 \\
 & x_2 & \geq & 0 \\
x_1 & & \in & \mathbb{N}
\end{array}
$$

6- Formulate the following problem and solve it using branch and bound algorithm.

An electricity producer should plan its hourly energy production to maximize its profits from selling energy during a planning horizon comprising 2 hours. Formulate and solve a mixed-integer linear programming problem to maximize the profits of the electricity producer, if

(a) The producer produces 5 energy units before the planning horizon.

(b) Hourly energy prices are 6 and 2 monetary units per energy unit.

(c) If running, the minimum energy production of the producer in each hour is 2 and the maximum one 10 energy units.

(d) Energy productions in two consecutive hours cannot differ in more than 4 energy units.

(e) Producer production cost is 3 monetary units per energy unit.